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PRIMORDIAL NUCLEOSYNTHESIS WITH GENERIC PARTICLES

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ABSTRACT. We discuss a revision of the standard model for big bang nucleosynthesis which allows for the presence of generic particle species. The primordial production of ^{4}He and D + ^{3}He is calculated as a function of the mass, spin degrees of freedom, and spin statistics of the generic particle for masses in the range $10^{-2} \lesssim m/m_e \lesssim 10^2$. The particular case of the Gelmini and Roncadelli majoron model for massive neutrinos is discussed.

1. INTRODUCTION

The standard model of big-bang nucleosynthesis[1] has proved an invaluable tool for placing constraints on various aspects of particle physics models.[2] With this in mind, we have developed a method which treats the effects of a generic particle species on primordial nucleosynthesis. In section 2 we review the pertinent features of the standard model and continue in section 3 to discuss the alterations to the standard model which are necessary to include generic particles. Section 4 contains the results for some generic particles and section 5 presents a discussion of a specific model for massive neutrinos.

2. A REVIEW OF THE STANDARD MODEL

The physics of big-bang nucleosynthesis consists of two distinct phases.[3] During the first phase, characterized by T > 1 MeV (throughout we use units so that $\hbar = c = k = 1$), the rate of weak interactions, $\Gamma_{wk} \propto G_F^2T^5$, is larger than the expansion rate, $\Gamma_{exp} \propto (G_N\rho)^{1/2} \propto G_N^{1/2}T^2$, and thus light neutrinos maintain thermal equilibrium with photons via $e^-e^+ \leftrightarrow \bar{\nu}_i\nu_i$. Nucleons maintain chemical equilibrium through $p\nu_e \leftrightarrow ne$, $n \leftrightarrow pe\bar{\nu}_e$, and $pe \leftrightarrow n\nu_e$ interactions and the neutron-to-proton ratio follows an equilibrium value,

$$(\frac{n}{p}) = \exp(-\frac{Q}{T}) \;;\; Q = m_n - m_p = 1.293 \text{ MeV}. \tag{1}$$

Neutrons and protons remain in equilibrium until weak interactions freeze out (i.e. $\Gamma_{wk} = \Gamma_{exp}$) at a temperature $T_F \simeq 0.7$ MeV. From this point, (n/p) slowly decreases due to non-equilibrium neutron decay until the onset of the second phase: nucleosynthesis. This second phase, characterized by $T \lesssim 0.1$ MeV, involves various nuclear interactions which proceed until $\Gamma_{nuclear} \lesssim \Gamma_{exp}$. For $T \gtrsim 0.1$ MeV, the formation of deuterium via $np \rightarrow d\gamma$ is suppressed by photodissociation due to the relatively low binding energy of the deuteron (2.2 MeV). The relative abundance of deuterons can be expressed

$$\frac{n_d}{n_N} \propto \eta \exp(\frac{2.2\text{MeV}}{T}), \tag{2}$$

where η is the nucleon-to-photon ratio. At $T \sim 0.1$ MeV, this bottleneck is circumvented and the various n,p,d reactions create D, ^{3}He, ^{3}H, ^{4}He, and ^{7}Li. The lack of stable mass 5 and 8 nuclei, Coulomb barriers, and the relatively large binding energy of ^{4}He result in most of the available neutrons being transformed into ^{4}He.

Primordial helium-4 production's dependence on these phases is described by three parameters: the nucleon-to-photon ratio η, the neutron half-life $\tau_{1/2}$, and the number of effective radiative degrees of freedom (usually parameterized as the number of light neutrino species N_ν). The amount of ^{4}He produced in the big-bang is dependent upon (n/p) at freeze out (Y_p = mass fraction of ^{4}He $\simeq 2(n/p)_F/[1+(n/p)_F]$). The earlier weak interactions freeze out, the greater $(n/p)_F$ and the larger Y_p. Thus the increase in Y_p with increasing $N_\nu(\sim\Gamma_{exp}^{1/2})$ and $\tau_{1/2}(\propto\Gamma_{wk}^{-1/2})$. Y_p also depends upon the efficiency of nuclear interactions at converting neutrons into ^{4}He. As the number density of photons decreases, the deuterium bottleneck breaks at higher temperatures, thus resulting in the increase of Y_p with increasing η.

The primordial production of D and ^{3}He also depends on the efficiency of ^{4}He production. Nearly all D and ^{3}He are processed into ^{4}He and thus their abundance depends upon the nuclear interaction rate - expansion rate interplay. Higher nucleon abundances (η) mean faster D-^{3}He depletion while a faster expansion rate ($N_\nu^{1/2}$) leads to an earlier freeze out of nuclear interactions, when D-^{3}He abundances are larger. Therefore, D and ^{3}He decrease with increasing η or $N_\nu^{-1/2}$.

3. ALTERATIONS FOR GENERIC PARTICLES

We have altered the original computer code of Wagoner[4] to correctly treat the additional physics of a generic particle species. By generic particle, we mean a particle which maintains good thermal contact with either photons or light neutrinos throughout nucleosynthesis. As such, the i^{th} generic particle can be described by a temperature T and has energy density and pressure

$$\rho_i(T) = \left[\frac{g_i}{2\pi^2}\int_{y_i}^{+\infty}\frac{\xi^2(\xi^2-y_i^2)^{1/2}d\xi}{e^{\xi+\theta_i}}\right]T^4 \tag{3}$$

$$P_i(T) = \left[\frac{g_i}{6\pi^2}\int_{y_i}^{+\infty}\frac{(\xi^2-y_i^2)^{3/2}d\xi}{e^{\xi+\theta_i}}\right]T^4 \tag{4}$$

where g_i is the number of spin states, (e.g. $g_e = 2$, $g_\nu = 1$, $g_\gamma = 2$, ...), $y_i = m_i/T$, and θ_i is +(-)1 for fermions (bosons) (in natural units $a = \pi^2/15$).

For T = 10 MeV, the universe is dominated by electrons, neutrinos, photons, and the generic particles. The entropy in a volume $R^3(t)$ ($R(t)$ is the FRW scale factor[5]) can be decomposed into two parts:

$$S_\gamma = \frac{R^3}{T_\gamma}\left[2\times(\rho_e+p_e) + \rho_\gamma + p_\gamma + \sum_i{}^{(\gamma)}(\rho_i+p_i)\right] \tag{5}$$

and

$$S_\nu = \frac{R^3}{T_\nu}\left[2\times N_\nu(\rho_\nu + p_\nu) + \sum_j{}^{(\nu)}(\rho_\nu + p_\nu)\right]. \tag{6}$$

The factor 2 results from consideration of particles and anti-particles, and the sums are over generic species which maintain good thermal contact with photons or light neutrinos. Up until T ~ 3 MeV[6], light neutrinos and photons maintain thermal equilibrium via neutral and charged current weak interactions and are described by the same temperature. Until this decoupling we have

$$S = S_\gamma + S_\nu = \text{constant} \qquad (T \geq T_{dec} = 3\ \text{MeV}) \tag{7}$$

Below T_{dec}, the decoupled light neutrinos maintain a thermal distribution described by T_ν and we have

$$\begin{aligned} S_\gamma &= \text{constant} = S_\gamma(T_\gamma=T_\nu=T_{dec}) \\ S_\nu &= \text{constant} = S_\nu(T_\nu=T_\gamma=T_{dec}) \end{aligned} \qquad (T \leq T_{dec}) \tag{8}$$

Using the above formalism we can describe the dynamical evolution of the universe through primordial nucleosynthesis as a function of T_γ[7]. The scale factor, $R(T_\gamma)$ can be expressed in terms of combinations of integrals like those of equations (3) and (4), as can the neutrino temperature $T_\nu(T_\gamma)$. The expansion rate as a function of T_γ follows from the field equation

$$\left(\frac{\dot{R}}{R}\right)^2 = \Gamma_{exp}^2 = \frac{8\pi G}{3}\rho \tag{9}$$

or

$$\int dt = \sqrt{\frac{3}{8\pi G}} \int \frac{dR}{R(T_\gamma)\sqrt{\rho(T_\gamma)}}, \tag{10}$$

The weak interaction physics is also a function of T_γ and T_ν through the phase space dependence of the initial and final states of the interactions $p\nu \leftrightarrow ne$, $pe \leftrightarrow n\nu$, and $n \leftrightarrow pe\nu$. We have supplemented Wagoner's code by numerically calculating the rates of these interactions, including radiative and Coulomb corrections[8], for given values of (T_γ, T_ν).

4. RESULTS FOR GENERIC PARTICLES

By assumption, a given generic particle species maintains good thermal contact through nucleosynthesis with either photons or light neutrinos and thus the physics of each generic species is completely specified by its mass, spin degrees of freedom, spin statistics, and annihilation mode. In figure 1, we show the ^4He (a) and $D+^3He$ (b) yields for primordial nucleosynthesis with generic particles coupled either to photons or light neutrinos, having g = 1,2, and in the mass range $10^{-2} \leq m/m_e \leq 10^2$. All calculations were done with $n_{10} = 3$, $\tau_{1/2}$ = 10.6 min, and $N_\nu = 3.0$. Horizontal dashed lines indicate standard model yields with the indicated number of light neutrino species.

The plots of Y_p and $D+^3He$ as a function of generic particle mass are most easily understood by considering three distinct regions characterized by m/m_e:

(I) $m/m_e \gtrsim 10^{1.75}$ $\quad$ ($m \gtrsim 30$ MeV)

(II) $1 \lesssim m/m_e \lesssim 10^{1.25}$ $\quad$ (0.5 MeV $\lesssim m \lesssim 10$ MeV)

(III) $m/m_e \lesssim 10^{-1}$ $\quad$ ($m \lesssim 50$ keV)

These regions are indicated in figure 1(a) and 1(b).

In all the graphs, we recover the standard model results for $m \gtrsim$ 30 MeV, since a particle of this mass will be present only in trivial abundances at the time of primordial nucleosynthesis. In addition, generic particles coupled to photons and having $m \lesssim 50$ keV overproduce ^4He and underproduce $D+^3He$ because these particles act as "pseudo-light neutrinos" increasing ρ_{TOT}, and because they dump most of their entropy after nucleosynthesis, necessitating a larger value of η during the bottleneck phase.[9] Generic particles coupled to neutrinos do not alter η and thus these particles with masses below 50 keV act only as extra light neutrino degrees of freedom (the g = 1 particle is a boson and contributes slightly more to the energy density than its fermion counterpart).

We need to consider the energy density as a function of T_γ to understand the Y_p and $D+^3He$ production for generic particles in the mass range 0.5 MeV - 10 MeV. The energy density for a universe containing a generic X particle which annihilates to photons can be written:

$$\rho_{X\to\gamma's} = a_1 T_\gamma^4 + a_2 \left(\frac{T_\nu}{T_\gamma}\right)^4_{X\to\gamma's} T_\gamma^4 + f(y_X^\gamma) T_\gamma^4, \tag{11}$$

where the first term accounts for electrons and photons, the second for light neutrinos, and the third for the X particles with $y_X^\gamma = m_X/T_\gamma$ and $f(y_X^\gamma)$ defined as T_γ^{-4} times equation (3). The energy density of the standard model is just

$$\rho_{STD} = a_1 T_\gamma^4 + a_2 \left(\frac{T_\nu}{T_\gamma}\right)^4_{STD} T_\gamma^4 . \tag{12}$$

We see that for a given T_γ we can have $\rho_{X\to\gamma's} < \rho_{STD}$, even though we have an extra X particle, provided

$$a_2 \left(\frac{T_\nu}{T_\gamma}\right)^4_{X\to\gamma's} + f(y_X^\gamma) < a_2 \left(\frac{T_\nu}{T_\gamma}\right)^4_{STD} . \tag{13}$$

in this case, $(T_\nu/T_\gamma)_{X\to\gamma's} \leq (T_\nu/T_\gamma)_{STD}$, and thus inequality (13) holds provided

$$f(y_X^\gamma) < a_2 \left[\left(\frac{T_\nu}{T_\gamma}\right)^4_{STD} - \left(\frac{T_\nu}{T_\gamma}\right)^4_{X\to\gamma's} \right] . \tag{14}$$

It is possible for particles in the mass range 0.5 MeV - 10 MeV to satisfy this inequality and the resulting $\rho_{X\to\gamma's} < \rho_{STD}$ causes an underproduction of both ^4He and $D+^3He$.

In a similar way, it is possible to have a generic particle coupled to neutrinos which has a mass so that $\rho_{X\to\nu's} > (\rho_{STD}$ + energy density of the massless equivalent of the X). For these generic particles, we observe overproduction of ^4He and $D+^3He$ relative to their extra light neutrino counterparts.

5. A SPECIFIC APPLICATION: HEAVY NEUTRINOS WITH MAJORONS

As a specific application of the generic particle approach, we have considered the effects of a heavy neutrino, like that described in majoron models[10], on primordial nucleosynthesis. In these models, neutrino masses are generated through the spontaneous breaking of a global B-L symmetry, and as such they contain a Goldstone boson, the majoron. The symmetry breaking is accomplished by expanding the Higgs sector to contain a triplet which couples to pairs of fermion doublet fields and acquires a non-zero vacuum expectation value (v.e.v.). The inclusion of the Higgs triplet introduces 6 massive scalars to the theory, 5 of which have masses on the order of the doublet v.e.v. (250 GeV) times a coupling constant. The remaining scalar picks up a mass similar to the heavy neutrino mass which in turn is proportional to the triplet v.e.v. The triplet v.e.v. is constrained by limits on majoron cooling of various astrophysical objects to be less than ~1 MeV.[11]

In majoron models, the dominant interaction is the majoron mediated annihilations of heavy to light neutrinos which has a cross section[12]

$$\sigma_{ann} = \frac{1}{32\pi} g_H^2 g_L^2 (\beta s)^{-1} , \tag{15}$$

where $g_{H(L)}$ is the heavy (light) neutrino-majoron coupling, β the c.m. neutrino velocity, and s the total c.m. energy squared. It can be shown[13] that such interconversions are sufficiently fast to keep the heavy neutrino in equilibrium with light neutrinos until after nucleosynthesis.

Thus we can treat these heavy neutrinos as generic neutrinos coupled to light neutrinos in order to determine their effect on primordial nucleosynthesis. For simplicity we take light Higgs boson and heavy neutrino to have the same mass and take $N_\nu = 2$, $\eta_{10} = 3$, and $\tau_{1/2} = 10.6$ min. The results are shown in figures 2(a) and 2(b). Noting that neutrinos less massive than about 1 MeV overproduce ^{4}He, one might try to decrease η to bring $Y_p = 25\%$, the accepted upper limit,[14] but we can't do this because that would make too much D+^{3}He.[15] Thus we conclude that majoron-type models with neutrino masses less than 1 MeV cannot reproduce observed primordial abundances. We also note that astrophysical limits on the triplet v.e.v. imply that $m_\nu \lesssim 1$ MeV and thus it appears that astrophysical/cosmological arguments can put severe constraints on the acceptable neutrino masses (i.e., $m_\nu \gtrsim 1$ MeV only) in majoron-type models, if not rule them out completely.[16]

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FOOTNOTES

(1) Wagoner, R. V., Fowler, W. A., and Hoyle, F., Ap. J. **148**, 3 (1967).

(2) For a review see, Yang, J., Turner, M. S., Steigman, G., Schramm, D. N., and Olive, K. A., Ap. J. **281**, 493 (1984).

(3) For current reviews of the standard model see Steigman, G. and Boesgaard, A. M. in Ann. Rev. Astron. Astrophys. (1985); Yang, J., etal. see Ref. 2.

(4) Wagoner, R. V., Ap. J. **179**, 343 (1973).

(5) For a discussion of the physics of the "early" universe, see, Weinberg, S., Gravitation and Cosmology, New York: Wiley and Sons, Inc. (1972).

(6) The actual neutrino decoupling temperature is different for each species. See, Dicus, D. A., Kolb, E. W., Gleeson, A. M., Sudarshan, E. C. G., Teplitz, V. L., and Turner, M. S., Phys. Rev. **D26**, 2694 (1982).

(7) Details of this technique can be found in Kolb, E. W., Turner, M. S., and Walker, T. P. (in preparation).
(8) See Ref. 6.
(9) In reality, the effect which generic particles have on η during nucleosynthesis must vanish for masses $\lesssim$ 100's of eV, since photons dumped later than this cannot thermalize.
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(11) Fukugita, M., Watamura, S., and Yoshimura, Phys. Rev. Lett. **48**, 1522 (1982); Glashow, S. L. and Manohar, A., Harvard Preprint HUTP-85/A031.
(12) Georgi, H. M., etal., see Ref. 9.
(13) See Ref. 7.
(14) Yang, J., etal., see Ref. 2.
(15) Ibid.
(16) Walker, T. P. (in preparation).